Comparative Channel Capacity Analysis of a MIMO Rayleigh Fading Channel with Different Antenna Spacing and Number of Nodes

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Abstract— *The presented research paper aims to analyze the channel capacity of a MIMO System in Rayleigh fading channel with different antenna nodes and spacing. It is argued that an implementable model is not readily available in the evaluated reference papers. As such a model is developed for simulating spatially correlated MIMO system in Rayleigh fading channel. To verify the developed model, a MATLAB code is devised to simulate the capacity of MIMO system for different numbers of antenna nodes and that is depicted in the comparative result MIMO systems have collected extensive attention in the new millennium due to its promise for a considerable increase in capacity, which is commonly considered to be a viable means for satisfying the ever-increasing demand for a higher data rate. The ground-breaking work by Telatar [1], Foschini and Gans [2] toward the end of last millennium not only provided some exciting results on capacity of multiple-antenna Gaussian channels but also stimulated a huge wave of enthusiasm toward various topics involving MIMO systems, including MIMO channel capacity, MIMO channel coding, space-time coding, etc. Among all lot of research and analysis is done in the area of channel capacity by modeling various channels and developing various algorithm.*

Keywords— MIMO, Rayleigh Fading, Spatially Correlated Capacity.

I. INTRODUCTION

After the pioneering work on MIMO systems was published by Telatar, Foschini and Gans in the late 1990's, MIMO capacity became a subject of significant research. Foschini [3] pointed out that the MIMO capacity could be substantially higher than that of a single-antenna system. When the concept of a multi-antenna system was originally introduced, no constraint was imposed on the input constellation. A Gaussian input was shown to achieve the capacity, or equivalently, to maximize the mutual information between input and output of a MIMO channel. However, it is impossible for a true Gaussian input to be realized in practice. The best one can do is approximate the Gaussian input with some sort of discrete inputs. The many currently available results on MIMO capacity are based on three exclusive assumptions: channel known at both the transmitter and receiver, channel known only at the receiver, and channel known at neither the transmitter nor the receiver. These MIMO systems can be studied from two different perspectives: one concerns performance evaluation in terms of error probability of practical systems, the other concerns the evaluation of the information theoretic Shannon capacity. The former can be obtained by simulation or analytically. For the later, the complementary cumulative distribution function of the capacity (sometimes called outage capacity) is derived. Generally mean capacity is derived for uncorrelated MIMO Rayleigh fading channels. The presented paper will show that MIMO systems in uncorrelated Rayleigh fading environments can potentially provide enormous System capacity. Further the simulation result show that by increasing the number of Antenna Nodes The capacity can further be improved. In this paper the fundamental concept and formulae is presented first. Later the simulated result is demonstrated. In our context we consider spatial correlated MIMO Channel and that is described in section 4.

This paper presents a study of existing literature on channel capacity of MIMO systems in a spatially correlated Rayleigh fading environment, being the most commonly studied channel model. Rayleigh fading model is used for analysis mainly because of its generality and applicability to the practical situation. A brief introduction of Rayleigh fading channel is presented in section 2. Further insights in the channel capacity formulae and the types of Rayleigh fading model is presented in section 3. MIMO capacity analysis of Rayleigh channel with spatial correlated environment is presented in section 4. Section 5 briefs about the description of the MIMO capacity MATLAB Simulation. The goal of this paper is to obtain the channel capacity of a MIMO system for spatially correlated Rayleigh fading channel and compare the results with interpretation. The conclusion is mentioned in section 7.

II. RAYLEIGH FADING CHANNEL

Rayleigh fading is a statistical model for the strong influence of a propagation environment on a radio signal, used by wireless communication devices. Rayleigh fading models consider that the magnitude of a signal that has passed through a transmission channel or medium will vary often and in a random manner, or fade, according to a Rayleigh distribution- the radial component of the addition of two uncorrelated Gaussian random variables. For wireless communications, the envelope of the received carrier signal is Rayleigh distributed; such a type of fading is called Rayleigh fading. This can be caused by multipath with or without the Doppler Effect. Rayleigh fading is most applied in situations when there is less or no dominant propagation along a line of sight between the transmitter and receiver. Presence of a dominant line of sight indicates that Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it finally reaches the receiver, just as in the case of MIMO System. According to the central limit theorem, if there is sufficiently too much scattering, the impulse response of the channel can be modelled well as a Gaussian process, not bothering about the distribution of the individual components. Absence of a dominant component to scatter clearly indicates that the process will have zero mean and phase evenly distributed between 0 and 2π radians. The envelope of the channel response will therefore be known as a Rayleigh distributed one.
Calling this random variable R, it will have a probability density function

$$P_R(r) = \frac{2r}{\Omega} e^{-\frac{r^2}{\Omega}}; r \geq 0; \quad \text{Where } \Omega = \left(R^2\right) \quad \textbf{(1)}$$

In the multipath case, when the dominant signal becomes weaker, such as in the non LOS case, the received signal is the sum of many components that are reflected from the surroundings. These independent scattered signal components that are reflected from the surroundings have different amplitudes and phases (time delays); then the in phase and quadrature components of the received signal can be assumed to be independent zero-mean Gaussian processes. This is derived from the central limit theorem, which states that the sum of a sufficient number of random variables approaches very closely to a normal distribution. When the mobile station moves, the frequency shift of each reflected signal component that arises from the Doppler Effect also has an influence on the fading. Very often, the gain and phase elements of a channel's distortion are represented as a complex number for mathematical convenience. In this scenario, Rayleigh fading is shown by the assumption that the real and imaginary parts of the channel response are well modeled by independent and identically distributed zero-mean Gaussian process so that the amplitude of the response is the sum of two such processes.

III. CHANNEL CAPACITY

There are several definitions of the MIMO channel capacity, depending on the scenario considered. The main differences between these definitions are due to the following. Channel state information (CSI): may be available at the receiver (Rx), transmitter (Tx), both or not at all (if CSI is available at Tx, water filling is possible). Ergodicity assumption: when channel is random, its capacity is random too; mean ergodic capacity may be defined if the ergodicity assumption is employed. Another possibility is to consider outage capacity. MIMO network capacity may also be defined when there are several users which interfere with each other. Since the arguments presented in this paper hold true for most definitions, we do not discuss in detail these differences. Among different notions of capacity, ergodic capacity and outage capacity are the two most often studied.

3.1 Ergodic Capacity

For ergodic capacity to be a legitimate characterization of a fading channel, the channel matrix H of a MIMO system; as a random process needs to be ergodic or changing fast enough. In other words, the fading has to be fast. This channel matrix H can be depicted by equation

$$y = Hx + n \quad \textbf{(2)}$$

Where, H is a complex Gaussian matrix, n represents complex Gaussian noise with a scaled identity matrix as its covariance matrix. Early research on MIMO ergodic capacity assumed no constraint on input signals. Under this assumption or lack thereof, Telatar [1], Foschini and Gans [2] separately obtained a similar form of ergodic capacity for channel model.

$$C_G = E\left[\log_2 \det\left(I_N + \frac{\gamma_s}{M} HH'\right)\right] \quad (3)$$

Where, the subscript G is used to emphasize the Gaussian characteristics of the capacity-achieving input signal.

3.2 Outage Capacity

The basic assumption for ergodic capacity is that the total transmission time is much longer than the coherence time of a fading channel. If this is not satisfied, as is the case in some real-time applications, e.g., speech transmission over wireless channels, the whole concept of ergodic capacity is no longer valid. In that case, we need to resort to a different definition of capacity, i.e., the outage capacity,

$$C_{out}(q) = \sup\left\{R \geq 0; P_r\left[I(x;y) < R\right] \leq q\right\} \quad (4)$$

Where, $q \in (0;1)$ is the so-called outage probability. Telatar also considered outage probability in [1],

$$P_{out}(R,P) = \inf_{\substack{Q\geq 0 \\ tr(Q)\leq P}} P_r\left[\log_2 \det\left(I_N + HQH'\right) < R\right] \quad (5)$$

Where, P is the power constraint, R is the supportable rate, $Q \geq 0$ means Q is a positive semi-definite matrix. This definition of outage probability is essentially the same as the definition of outage capacity in equation 4.

IV. MIMO CAPACITY OF RAYLEIGH CHANNEL WITH SPATIAL CORRELATED FADING CHANNEL

The Rayleigh channel assumes flat fading in the space, time, and frequency domains. However, the signal components arriving at the receiver may experience correlation due to the limited distance of the antenna elements. As a result, the use of HW as the channel matrix is inappropriate. In order to include the correlation effect the following equation is used [4].

$$vec(H) = R^{\frac{1}{2}} vec(H_w) \quad (6)$$

Where, vec (H), denotes a vector made by the columns of H, and R is the covariance matrix of the channel which is given by

$$R = E\left\{vec(H)vec(H)^H\right\} \quad (7)$$

The analysis can be simplified with the use of the channel matrix of equation 8

$$H = R_R^{\frac{1}{2}} H_W R_T^{\frac{1}{2}} \quad (8)$$

Where, R_R is the reception correlation matrix and R_T is the transmission correlation matrix. Equation 8 is derived by equation 6 under the assumption that matrix R_R and R_T remain unchanged, regardless of the transmitting and receiving elements, respectively. A very interesting result [5] is depicted in the figure 1, which illustrates the capacity for different antenna configurations and inter element spacing distances, d in wavelengths. Figure 1 proves the great effect of correlation to the MIMO channel capacity. We can easily notice that the uncorrelated channel $(d = 0.5\lambda)$ offers high capacity performance in comparison to rest cases. Specifically, the ergodic capacity of the (4, 4) uncorrelated channel is about 13 bits/sec/Hz greater than the fully correlated one. So as the distance between the antenna elements decreases, the capacity decreases too. The reason lies in the correlation increase with the decrease of the inter element distance. Correlation between the transmitted and received signals decreases the independent propagation paths and, as a result, decreases the information transferred. Also, we notice that the (4, 4) MIMO channel achieves higher capacity compared to the (2, 2) channel, under any correlation conditions.

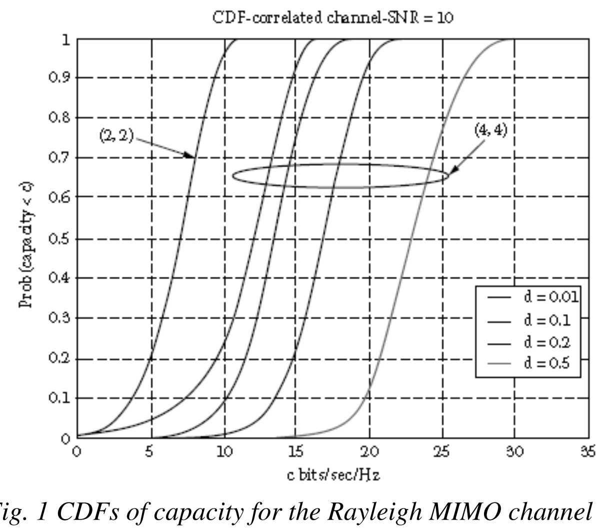

Fig. 1 CDFs of capacity for the Rayleigh MIMO channel with spatial fading correlation.

V. SIMULATION DESCRIPTION

To demonstrate the relation of the MIMO channel capacity with respect to spacing and the number of nodes we have referred [6]. This implementation will verify the earlier analysis, which is reviewed in the paper. This section describes the content of the MATLAB® implementation of the MIMO radio channel. The package contains two directories Correlation_Multiple_Cluster and UMTS_Testbed where one can find the MATLAB® scripts enabling to

- Derive the spatial correlation properties of a Uniform Linear Array (ULA) impinged by a Gaussian PAS, where the waves are gathered in a single or in multiple clusters.
- Simulate a MIMO radio channel at link-level in compliance with 3GPP specifications [7]. The simulated model is fully described in [8][9].

VI. SIMULATION RESULT

Here we present two tables comprises of system capacity of a Rayleigh fading channel. The first table carries the capacities for 2x2 MIMO channel with different antenna spacing. In the second table we kept the antenna spacing constant but have changed the number of nodes.

TABLE 1 - MIMO SETUP FOR DIFFERENT ANTENNA SPACING

3GPP Case	Spacing between Nodes(λ)	No of Antenna Nodes	Type of Channel	System Capacity
1	0.01	2	Rayleigh	3.9886
1	0.5	2	Rayleigh	2.3065
1	2	2	Rayleigh	2.5790

TABLE 2 - MIMO SETUP FOR DIFFERENT NUMBERS OF ANTENNA NODES

3GPP Case	Spacing between Nodes(λ)	No of Antenna Nodes	Type of Channel	System Capacity
1	0.5	2	Rayleigh	2.3065
1	0.5	3	Rayleigh	4.1171
1	0.5	4	Rayleigh	4.6200

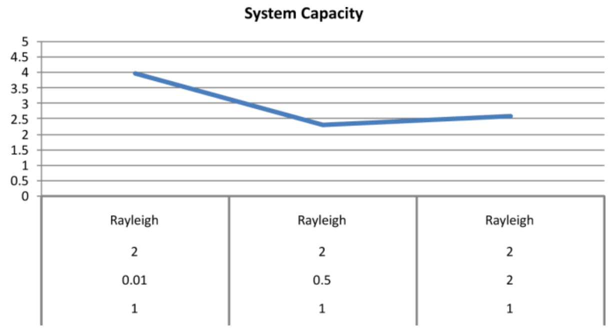

Fig. 2 Graphical representation of the channel capacity according to table 1

VII. CONCLUSION

As the earlier statement in section 4, which is depicted from [5] brief us that, for same number of nodes the capacity decreases with increase in antenna spacing. That is due to decrease in correlation effect with increase in spacing. The correlation is assumed up to $(d = 0.5\lambda)$. This statement is only justified with more number of antenna nodes. In our case the system is having 2x2 arrays. Here we can observe in table 1 that for correlation case the capacity is far greater than the uncorrelated one. Here the spacing 2λ is far greater than the threshold limit. That is why treated as uncorrelated one. Further in table 2, an interesting result justifies the earlier research and simulation that the channel capacity increases with increase in number of nodes for given distance. Figure 2 shows the graphical representation of the results.

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